Observing 2-channel Kondo physics in a carbon nanotube single-electron transistor

2004 J. Phys.: Condens. Matter 16 749

(http://iopscience.iop.org/0953-8984/16/6/006)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) 749–755 PII: S0953-8984(04)70778-X

Observing 2-channel Kondo physics in a carbon nanotube single-electron transistor

Eugene H Kim[1,2], **Germàn Sierra**[3] **and C Kallin**[2]

[1] Department of Physics, University of Toronto, Toronto, ON, M5S 1A7, Canada
[2] Department of Physics and Astronomy, McMaster University, Hamilton, ON, L8S-4M1, Canada
[3] Instituto de Matemàticas y Fìsica Fundamental, CSIC, 28006 Madrid, Spain

Received 20 October 2003
Published 30 January 2004
Online at stacks.iop.org/JPhysCM/16/749 (DOI: 10.1088/0953-8984/16/6/006)

Abstract
Recently, Coulomb blockade physics was observed at room temperature in a carbon nanotube single-electron transistor (Postma *et al* 2001 *Science* **293** 76). In this work, we suggest that these devices may be promising for observing 2-channel Kondo physics, and allow for a detailed investigation of the 2-channel Kondo fixed point. Experimental signatures of the Kondo effect in these systems are discussed.

Since the seminal work of Nozieres and Blandin [1], the multichannel Kondo model, and in particular the 2-channel Kondo (2CK) model, has received an enormous amount of attention [2]. In this model, a single magnetic impurity with spin s is coupled to k channels of conduction electrons ($k > 1$). Remarkably, if $k > 2s$ the system exhibits non-Fermi-liquid behaviour at low energies. Since such non-trivial physics arises from such a (seemingly) simple model, it is not surprising this model would be of interest to so many people.

Unfortunately, to date there has been no conclusive observation of 2CK (or, more generally, multichannel Kondo) physics. However, this is not for lack of trying [2]. Probably the most promising system for observing 2CK physics was the tunnelling 2-level system [2]. Indeed, the conductance signal observed in ballistic metal point contacts are consistent with 2CK scattering from tunnelling 2-level systems in the constriction [3]. However, the interpretation in terms of 2CK physics is still unsettled [3–6].

Recently, quantum dots have provided a renewed excitement about the Kondo effect [7]. These systems are ideal for studying Kondo physics, because there are many parameters which can be controlled. Therefore, many aspects of the Kondo effect can be probed. In recent work [8], a single-electron transistor (SET) was fabricated by introducing two buckles in series in a long single-wall carbon nanotube. The two buckles define a small island (i.e. a 'quantum dot') within the nanotube (see figure 1 of [8]). Using this device, the authors of [8] observed Coulomb blockade physics at room temperature. Moreover, they found that the conductance had a power-law temperature dependence, consistent with a Luttinger liquid model for the

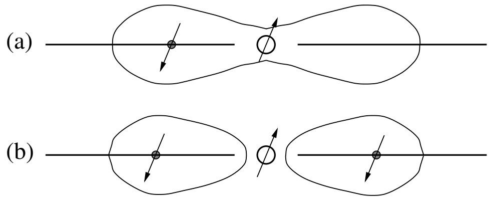

Figure 1. (a) Screening cloud for the case of non-interacting leads. The low energy physics is described by the 1-channel Kondo fixed point. (b) Screening cloud with (strong) interactions in the leads. The low energy physics is described by the 2CK fixed point.

leads. In this work, we suggest that these devices could be promising for studying Kondo physics, and in particular, for observing the physics of the elusive 2CK fixed point.

The carbon nanotube SET is promising for several reasons. It has been shown that a quantum dot coupled to Luttinger liquid leads will flow to the 2CK fixed point, provided the Luttinger parameter is small enough [9, 10]. Carbon nanotubes are the only material we are aware of where the Luttinger parameter is small enough for this to happen (see below for further discussion). Moreover, the device of [8] allows for the possibility of achieving high Kondo temperatures. This is because the charging energy (U_0) and single-particle level-spacing (δ) of this device, which set the scale for the Kondo temperature, are so large: $U_0 \approx 82$ meV ($\approx$951 K) and $\delta \approx 41$ meV ($\approx$476 K). These values should be compared to typical values in semiconductor devices, $U_0 \approx 2$ meV ($\approx$23 K) and $\delta \approx 400$ μeV ($\approx$5 K), where Kondo temperatures as high as $T_K \approx 3$ K were achieved [11]. It is worth noting that the Kondo temperature is what casts the most serious doubt on 2CK physics in tunnelling 2-level systems. More specifically, it has been shown that tunnelling 2-level systems will have Kondo temperatures which are unattainably low, making the observation of 2CK physics unlikely [5].

We begin our discussion by recalling some facts about carbon nanotubes [12]. As is well known, these materials consist of a sheet of graphite rolled into a cylinder. Interestingly, 'armchair' nanotubes (such as those used in [8]) possess two one-dimensional bands of gapless excitations [13]. These two bands (which we label as *band-c* and *band-d*) disperse with the same velocity, and determine the low energy electronic properties. Since carbon nanotubes are essentially one-dimensional, the interactions between electrons have a pronounced effect [14]. For isolated single-wall nanotubes, the Coulomb interaction is unscreened. This long ranged interaction causes the system to behave as a Luttinger liquid at all temperatures [15, 16]. Interbranch (i.e. backscattering) interactions, on the other hand, are weak [15]. In principle, these (backscattering) interactions will open a spin gap [17]. However, the spin gap is expected to be very small. Indeed, in [8], Luttinger liquid behaviour was observed down to 4 K with no sign of a spin gap, implying that the spin gap energy scale must be considerably lower than 4 K. Therefore, we believe it is safe to ignore the effects of the spin gap (if necessary, a small magnetic field could be applied to break the spin gap).

In [8], an SET was fabricated by creating a small island within a long single-wall carbon nanotube. As this device was made by introducing buckles in a nanotube, it will probably be difficult to control. In particular, it will probably be difficult to control the couplings between the leads and the island. Considering the enormous advances in experiments on nanoscale systems, it is not unreasonable to assume that a modified form of this device can be fabricated, where the various couplings can be controlled. In the following, we will assume this to be the case; we will show that such a device could allow for a detailed investigation of the 2CK fixed point.

Being interested in the low energy properties of the system, we focus on the uppermost level of the island and model it as an Anderson impurity. The Hamiltonian we consider is

$$H_{\text{island}} = \varepsilon_0 \sum_s n_s^f + U_0 n_\uparrow^f n_\downarrow^f - \frac{h_0}{2}(n_\uparrow^f - n_\downarrow^f) - \sum_{\substack{\lambda=c,d \\ s=\uparrow,\downarrow}} (t_{1\lambda}\psi^\dagger_{1,\lambda,s}(0) + t_{2\lambda}\psi^\dagger_{2,\lambda,s}(0)) f_s + \text{h.c.}, \tag{1}$$

where $\psi_{i,\lambda,s}$ destroys an electron with spin-s in lead-i ($i = 1, 2$) and band-λ ($\lambda = c, d$); f_s destroys an electron with spin-s on the island; $n_s^f = f_s^\dagger f_s$; ε_0 is the energy level of the island, which can be controlled by a gate voltage; U_0 is the charging energy; h_0 is the magnetic field; $t_{i\lambda}$ is the matrix element for an electron to tunnel to the island from band-λ in lead-i. It is useful to introduce *bonding* and *antibonding* combinations

$$\begin{aligned} \psi_{i,b,s} &= (t_{ic}\psi_{i,c,s} + t_{id}\psi_{i,d,s})/\sqrt{N_i}, \\ \psi_{i,a,s} &= (t_{id}\psi_{i,c,s} - t_{ic}\psi_{i,d,s})/\sqrt{N_i}, \end{aligned} \tag{2}$$

with $N_i = t_{ic}^2 + t_{id}^2$. In terms of these operators, we see that only the bonding combinations couple to the island. Being interested in the Kondo regime, we integrate out charge fluctuations on the island. Working to second order in perturbation theory [18], we arrive at the effective Hamiltonian

$$\begin{aligned} H_{\text{int}} = \boldsymbol{\tau} \cdot \frac{\sigma_{s,s'}}{2} (J_1\, \psi^\dagger_{1,b,s}(0)\psi_{1,b,s'}(0) + 1 \to 2) \\ + J_{12}\boldsymbol{\tau} \cdot \frac{\sigma_{s,s'}}{2}(\psi^\dagger_{1,b,s}(0)\psi_{2,b,s'}(0) + \text{h.c.}) - h_0\tau_z, \end{aligned} \tag{3}$$

where τ is the spin operator for the electron on the island, and

$$J_{ij} = (2t_i t_j/\varepsilon_0)U_0/(U_0 - \varepsilon_0),$$

with $J_i \equiv J_{ii}$. It is important to note that $J_{ij} > 0$. It should also be noted that in equation (3) we have not displayed the potential scattering terms [18] which were generated. For the system considered in this work, these terms have a very small effect and can be ignored [10].

The dynamics of the leads is described by the Hamiltonian $H_{\text{leads}} = H_{\text{lead}-1} + H_{\text{lead}-2}$, where $H_{\text{lead}-i} = H_i^0 + H_i^1$ is the Hamiltonian for lead-i with [15]

$$\begin{aligned} H_i^0 &= -\mathrm{i}v_F \sum_{\lambda,s} \int_{-\infty}^0 \mathrm{d}x\, (\psi^\dagger_{\mathrm{R},i,\lambda,s}\partial_x\psi_{\mathrm{R},i,\lambda,s} - \mathrm{R} \to \mathrm{L}) \\ H_i^1 &= U \int_{-\infty}^0 \mathrm{d}x \left(\sum_{\lambda,s} \psi^\dagger_{\mathrm{R},i,\lambda,s}\psi_{\mathrm{R},i,\lambda,s} + \psi^\dagger_{\mathrm{L},i,\lambda,s}\psi_{\mathrm{L},i,\lambda,s} \right)^2. \end{aligned} \tag{4}$$

In the above equation, $\psi_{\mathrm{R},i,\lambda,s}$ ($\psi_{\mathrm{L},i,\lambda,s}$) is the right (left) moving component of $\psi_{i,\lambda,s}$. Furthermore, we have followed [15] and taken the Coulomb interaction to be screened beyond some long distance; U is the effective strength of this interaction. In the previous paragraph, we saw that only the bonding combination of the fermion fields (equation (2)) couples to the impurity. Fortunately, we can express the Hamiltonian of the leads in terms of the bonding and antibonding operators as well. In terms of these operators, the Hamiltonian has the same form as equation (4), except the labels c and d are replaced everywhere by b and a.

In what follows, we will make extensive use of the boson representation. To do so, the electron operator is written as $\psi_{\mathrm{R/L},i,\lambda,s} \sim \mathrm{e}^{\pm\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{R/L},i,\lambda,s}}$ where the chiral fields, $\phi_{\mathrm{R},i,\lambda,s}$ and $\phi_{\mathrm{L},i,\lambda,s}$, are related to the usual Bose field $\phi_{i,\lambda,s}$ and its dual field $\theta_{i,\lambda,s}$ by $\phi_{i,\lambda,s} = \phi_{\mathrm{R},i,\lambda,s} + \phi_{\mathrm{L},i,\lambda,s}$ and $\theta_{i,\lambda,s} = \phi_{\mathrm{R},i,\lambda,s} - \phi_{\mathrm{L},i,\lambda,s}$. It will also prove useful to form *charge* and *spin* fields $\phi_{i,\lambda,\rho/\sigma} = (\phi_{i,\lambda,\uparrow} \pm \phi_{i,\lambda,\downarrow})/\sqrt{2}$, and then form the combinations $\phi_{i,\rho^\pm} = (\phi_{i,b,\rho} \pm \phi_{i,a,\rho})/\sqrt{2}$

describing *total* and *relative* charge fluctuations in lead-i. In terms of these variables, the Hamiltonian for lead-i is

$$H_{\text{lead}-i} = \frac{v_{\rho^+}}{2}\int_{-\infty}^{0} \mathrm{d}x\, K_{\rho^+}(\partial_x\theta_{i,\rho^+})^2 + \frac{1}{K_{\rho^+}}(\partial_x\phi_{i,\rho^+})^2 + \frac{v_F}{2}\int_{-\infty}^{0} \mathrm{d}x\, (\partial_x\theta_{i,\rho^-})^2 + (\partial_x\phi_{i,\rho^-})^2 + \frac{v_F}{2}\sum_{\lambda=b,a}\int_{-\infty}^{0} \mathrm{d}x\, (\partial_x\theta_{i,\lambda,\sigma})^2 + (\partial_x\phi_{i,\lambda,\sigma})^2, \tag{5}$$

where $K_{\rho^+} = 1/\sqrt{1+8U/(\pi v_F)}$ and $v_{\rho^+} = v_F/K_{\rho^+}$. Experimentally, it has been found that $0.19 \leqslant K_{\rho^+} \leqslant 0.26$ for single-wall carbon nanotubes [8]. Finally, to analyse the physics it will prove useful to unfold the system, and work solely in terms of right moving fields [19].

As our interest is in the physics of the 2CK fixed point, we will assume the system is close to this fixed point in what follows. However, before proceeding with our analysis, it is useful to review how 2CK physics arises. Recall that for the case of non-interacting leads, the system flows to the 1-channel Kondo fixed point. This occurs because an electron in the symmetric orbital centred about the island will screen the electron on the island (see figure 1(a)). However, with interactions in the leads (and small enough Luttinger parameter), tunnelling between leads is suppressed [9, 10]. Note that in order to form a screening cloud in the symmetric orbital centred about the island, one must be able to freely tunnel between the two leads. However, if tunnelling is suppressed, heuristically, this forces two screening clouds to form. Hence, the impurity is overscreened; the system flows to the 2CK fixed point (see figure 1(b)).

To analyse the physics, we first start with equation (3). Using the renormalization group (RG), we trace the RG flows of the parameters as the system flows close to the 2-channel Kondo fixed point: $J_1 + J_2 = \mathcal{O}(1)$; $J_1 - J_2 \ll \mathcal{O}(1)$; $J_{12} \ll \mathcal{O}(1)$; $h_0 \ll \mathcal{O}(1)$. (Note that we are assuming $J_1 \approx J_2$, so that the system does, in fact, flow close to the 2CK fixed point.) Then, to analyse the physics near the 2CK fixed point, we follow [20] and form combinations of the fields in the two leads: $\phi_{\mathrm{R},c}$, $\phi_{\mathrm{R},sp}$, $\phi_{\mathrm{R},f}$, and $\phi_{\mathrm{R},sf}$. Next, we perform the unitary transformation, $U = \exp(\mathrm{i}\sqrt{4\pi}\tau^z\phi_{\mathrm{R},sp}(0))$, which ties a spinon from the leads to the island. Finally, we introduce new fermion fields, $d \sim \tau^-$ and $X \sim \mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{R},sf}}$. Upon performing these transformations, H_{int} becomes

$$H_{\text{int}} = v_F\lambda_+(d^\dagger - d)(X^\dagger(0) + X(0)) + v_F\lambda_-(d^\dagger + d)(X^\dagger(0) - X(0)) - v_F\lambda_h(d^\dagger d - 1/2) + v_F g(d^\dagger + d)(\mathrm{e}^{-\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{R},f}(0)} - \mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{R},f}(0)}), \tag{6}$$

where $\lambda_+ \sim J_1' + J_2'$, $\lambda_- \sim J_1' - J_2'$, $g \sim J_{12}'$, and $\lambda_h \sim h_0'$. (J_1', J_2', J_{12}', and h_0' are the renormalized values of the parameters near the 2CK fixed point.) Note that in equation (6), we have displayed only the most relevant operators.

A few words are in order about equation (6). To begin with, the λ_+ term sets the 2CK energy scale; the g, λ_-, and λ_h terms are perturbations about the 2CK fixed point. The g term has dimension $(1 + 1/K_{\rho^+})/4$, and is relevant for $K_{\rho^+} > 1/3$. Hence, this term is irrelevant for the system we are considering. Both the λ_- and λ_h terms have dimension 1/2 and are relevant. If these terms are absent, the zero temperature fixed point would be the 2CK fixed point. However, nonzero λ_- and/or λ_h drives the system away from the 2CK fixed point. It should be noted that the dimensions of the g, λ_-, and λ_h terms are properties of the 2CK fixed point. In particular, they are different from the dimensions near the ultraviolet fixed point; they can, in fact, be observed experimentally (see below).

We begin by considering the case where the system flows to the 2CK fixed point: $\lambda_-, \lambda_h = 0$. Signatures of the 2CK fixed point can be observed in conductance measurements.

Using the Golden Rule, we find (for $K_{\rho^+} < 1/3$)

$$G/G_0 = \frac{1}{\Gamma(\beta)}\left(\frac{T}{T_{\mathrm{K}}}\right)^{\beta-2}\cosh\left(\frac{eV}{2T}\right)\left|\Gamma\left(\frac{\beta}{2}+\mathrm{i}\frac{eV}{2\pi T}\right)\right|^2 \times\left\{1+\frac{2}{\pi}\tanh\left(\frac{eV}{2T}\right)\mathrm{Im}\left[\psi\left(\frac{\beta}{2}+\mathrm{i}\frac{eV}{2\pi T}\right)\right]\right\}. \tag{7}$$

In equation (7), $G_0 = (2e^2/h)(g')^2/(2\pi)$; $\beta = (1/2)(1+1/K_{\rho^+})$; $\Gamma(x)$ is the gamma function, and $\psi(x)$ is the psi function [21]. Notice that equation (7) exhibits ω/T-scaling. This occurs because the 2CK fixed point is a non-trivial scale-invariant fixed point. Moreover, in linear response ($V \to 0$) $G \sim T^{\beta-2}$. This temperature dependence is a property of the 2CK fixed point, and reflects the dimension of the g term in equation (6). With no Kondo effect, one expects $G \sim T^{\alpha-2}$ where $\alpha = (1/2)(3+1/K_{\rho^+})$. This behaviour is what was observed in [8]. Notice that the exponent is smaller near the 2CK fixed point than when there is no Kondo effect. In other words, tunnelling is enhanced near the 2CK fixed point, as compared to when there is no Kondo effect. This can be understood rather easily. It is well known that in a Luttinger liquid, spin and charge separate. Also, only electrons can tunnel between leads; the fractionalized excitations of the Luttinger liquid are unable to tunnel between leads. However, in order for an electron to tunnel, spin and charge must first recombine to form an electron; then the electron can tunnel. An important property of the 2CK fixed point is that a spinon from the leads is tied to the island (see the discussion above equation (6)). Since a spinon is tied to the island, it is easier for spin and charge to recombine. Hence, interlead tunnelling increases, and the exponent decreases.

Since $G \to 0$ as $T \to 0$, it may appear the system is behaving simply as two (semi-infinite) decoupled nanotubes. However, this is not correct. This is most apparent if one considers the spin conductance—two decoupled nanotubes would have vanishing spin conductance (for $T \to 0$); the 2CK fixed point has perfect spin conductance [10]. This can also be understood rather easily. A heuristic picture of 2CK physics (with a spin-1/2 impurity) is that the impurity traps two electrons, which are aligned antiparallel to the impurity. The impurity plus the two trapped electrons gives a net effective spin-1/2 impurity. This effective spin-1/2 impurity then goes on to trap two electrons, which are aligned antiparallel to the effective impurity. This procedure continues *ad infinitum* [1]. In doing this, the two semi-infinite leads are essentially joined together to form a single infinite lead, as far as spin excitations are concerned. Recall that, for the case of non-interacting leads, the system flows to the one-channel Kondo fixed point; the signature of the Kondo effect is perfect conductance through the island [7]. Essentially, the Kondo effect has joined two semi-infinite leads together to form a single infinite lead. Here, a similar effect occurs, except the two leads are joined together only in the spin sector (and not the charge sector). Unfortunately, the spin conductance is difficult to measure. However, this should be observable in thermal conductance measurements. More specifically,

$$\kappa \to \kappa_0 \equiv \frac{\pi^2}{3h}T \qquad \text{as } T \to 0,$$

if the system flows to the 2CK fixed point (κ_0 is the value for perfect thermal conductance).

Now we consider the case where $\lambda_-, \lambda_h \neq 0$. These terms are relevant perturbations to the 2CK fixed point. Moreover, as mentioned above, both of these terms have dimension 1/2 near the 2CK fixed point. The λ_- term drives the system to the 1-channel Kondo fixed point, where the electron on the island forms a singlet with the electrons in the lead with the larger exchange coupling [1, 22]. The λ_h term drives the system to a fixed point where the electron on the island is spin polarized; spin-flip processes are energetically costly, and the electron on the island behaves as a potential scatterer [22]. The energy scale at which 2CK behaviour will

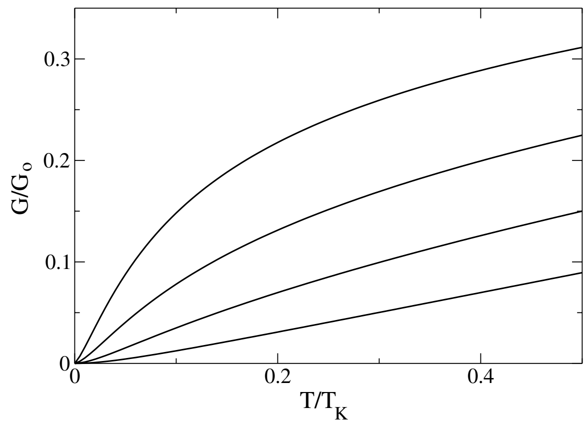

Figure 2. G/G_0 versus T/T_K. $K_\rho = 0.29, 0.26, 0.23, 0.2$ in order from the top to the bottom curve. Γ_- and Γ_h were taken to be $\Gamma_- = 0.07$ and $\Gamma_h = 0.1$.

no longer be observable is determined by the values of λ_- and λ_h. The effects of these terms can be most directly observed in the spin conductance. However, as mentioned above, since the spin conductance is difficult to measure, the thermal conductance is the most promising place to observe these effects. As charge transport is suppressed, the thermal conductance will be dominated by spin. Computing the thermal conductance [23] due to spin, we find

$$\kappa/\kappa_0 = \left(\frac{3}{4\pi^2}\right)\left(\frac{T_K}{T}\right)^3 \int \mathrm{d}x\ \mathrm{sech}^2\left(\frac{xT_K}{2T}\right)\frac{x^4(1-\Gamma_-)^2}{(x^2-\Gamma_h-\Gamma_-)^2+x^2(1+\Gamma_-)^2}, \tag{8}$$

where $\Gamma_- \sim \lambda_-^2$ and $\Gamma_h \sim \lambda_h^2$.

To better understand equation (8), let us consider the case $\Gamma_h = 0$, $\Gamma_- \neq 0$. (The case $\Gamma_- = 0$, $\Gamma_h \neq 0$ gives similar results.) Then, for $0 \ll T \ll T_K$, equation (8) gives

$$\kappa/\kappa_0 = 1 - \Gamma_-^2\frac{3}{\pi^2}\left(\frac{T_K}{T}\right)^2.$$

The $1/T^2$ correction to the thermal conductance is a property of the 2CK fixed point, and reflects the dimension of the λ_- term. However, as can be seen from equation (8), the $1/T^2$ temperature dependence will be changed as we go to even lower temperatures, since at those energies the system is far away from the 2CK fixed point.

The effects of λ_- and λ_h can also be observed in the (charge) conductance. In linear response, we find

$$G/G_0 = \frac{1}{\Gamma(\beta)}\left(\frac{T}{T_K}\right)^{\beta-2}\int\frac{\mathrm{d}x}{2\pi}\,\mathrm{sech}\left(\frac{xT_K}{2T}\right)$$
$$\times\left|\Gamma\left(\frac{\beta}{2}+\mathrm{i}\frac{xT_K}{2\pi T}\right)\right|^2\frac{\Gamma_-(1+x^2)+\Gamma_h}{(x^2-\Gamma_h-\Gamma_-)^2+x^2(1+\Gamma_-)^2}. \tag{9}$$

G/G_0 versus T/T_K is plotted in figure 2 for several values of K_{ρ^+}. Note that G shows 2CK behaviour ($G \sim T^{\beta-2}$) for $\Gamma_h, \Gamma_- \ll T \ll T_K$. However, for $T < \Gamma_-$ and/or $T < \Gamma_h$, the system is far from the 2CK fixed point, and the temperature dependence is modified from its 2CK behaviour.

In conclusion, carbon nanotube SETs [8] may be promising for observing 2CK physics, and could allow for a detailed investigation of the 2CK fixed point. It is worth noting that generalizations of this device could allow for the study of other related phenomena. For example, introducing two islands in the nanotube could allow one to study two-impurity Kondo physics, or, more generally, the properties of coupled quantum dots. Moreover, short carbon nanotubes have been shown to exhibit properties characteristic of nanoscale conductors. By fabricating a similar device in a short nanotube, one could study interesting finite-size effects. In particular, this system may allow for the elusive Kondo cloud to finally be observed [24].

Acknowledgments

EHK is grateful to H Paik for bringing [8] to his attention. This work was supported by the NSERC of Canada (EHK and CK), Materials and Manufacturing of Ontario (EHK and CK), and the Spanish grant PB98-0685 (GS).

References

[1] Nozieres P and Blandin A 1980 *J. Physique (Paris)* **41** 193
[2] For a review, see Cox D L and Zawadowski A 1998 *Adv. Phys.* **47** 599
[3] Ralph D C and Buhrman R A 1992 *Phys. Rev. Lett.* **69** 2118
Ralph D C, Ludwig A W W, von Delft J and Buhrman R A 1994 *Phys. Rev. Lett.* **72** 1064
[4] Wingreen N S, Altshuler B L and Meir Y 1995 *Phys. Rev. Lett.* **75** 769
[5] Aleiner I L *et al* 2001 *Phys. Rev. Lett.* **86** 2629
[6] Zawadowski A and Zaránd G 2000 *Preprint* cond-mat/0009283
[7] For a review, see Kouwenhoven L and Glazman L 2001 *Phys. World* **14** (1) 33
[8] Postma H W Ch *et al* 2001 *Science* **293** 76
[9] Fabrizio M and Gogolin A O 1995 *Phys. Rev.* B **51** 17827
[10] Kim E H 2001 *Preprint* cond-mat/0106575
[11] See, for example, Goldhaber-Gordon D *et al* 1998 *Phys. Rev. Lett.* **81** 5225
[12] For reviews, see Dekker C 1999 *Phys. Today* **52** (5) 22
McEuen P L 2000 *Phys. World* **13** (6) 31
[13] Hamada N *et al* 1992 *Phys. Rev. Lett.* **68** 1579
Mintmire J W *et al* 1992 *Phys. Rev. Lett.* **68** 631
Saito R *et al* 1992 *Appl. Phys. Lett.* **60** 2204
[14] See, for example, Solyom J 1979 *Adv. Phys.* **28** 201
[15] Kane C L, Balents L and Fisher M P A 1997 *Phys. Rev. Lett.* **79** 5086
[16] Egger R and Gogolin A O 1997 *Phys. Rev. Lett.* **79** 5082
[17] See, for example, Lin H-H, Balents L and Fisher M P A 1997 *Phys. Rev.* B **56** 6569 and references therein
[18] Schrieffer J R and Wolff P A 1966 *Phys. Rev.* **149** 491
[19] Eggert S and Affleck I 1992 *Phys. Rev.* B **46** 10866
[20] Schiller A and Hershfield S 1995 *Phys. Rev.* B **51** R12896
Majumdar K, Schiller A and Hershfield S 1998 *Phys. Rev.* B **57** 2991
[21] Gradshteyn I S and Ryzhik I M 1994 *Table of Integrals, Series, and Products* (San Diego, CA: Academic)
[22] Affleck I *et al* 1992 *Phys. Rev.* B **45** 7918
[23] Sivan U and Imry Y 1986 *Phys. Rev.* B **33** 551
[24] Simon P and Affleck I 2002 *Phys. Rev. Lett.* **89** 2066